Math 62 12.6

Math 72 10.3 - 1st

Solving Systems of Nonlinear Equations
A.K.A "nonlinear systems".

Objectives
1) Review like terms
2) Solve systems by substitution
3) Solve systems by elimination

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Review of like terms:

- <u>terms</u> are separated by add or subtract only

<u>ex 1</u> $x^2 + x - 3$ has 3 terms $x^2$
$x$
$-3$

<u>ex 2</u> $x^2 - 3y^2$ has 2 terms $x^2$
$-3y^2$

- <u>like terms</u> have the same variables and the same exponents on those variables

<u>ex 3</u> $-3x^2$ and $2x^2$ are like terms

<u>ex 4</u> $-3x^2$ and $2x$ are not like terms

- Like terms can be combined (add/subtract)
"adding apples to apples"

<u>ex. 5</u> $-3x^2 + 2x^2 = (-3+2)x^2 = \boxed{-x^2}$

add coefficients

Type of term does not change, exponent unchanged

<u>ex 6:</u> Simplify $-2x^2 + 3x + 2x^2 - 7x$

$x^2$ like terms

$x$ like terms

$= (-2+2)x^2 + (3-7)x$

$= 0x^2 - 4x$

$= \boxed{-4x}$

zero times anything is zero.

Math 70 M6 5/e 10.3 Solving Nonlinear Systems of Equations

Objectives
1) Solve a nonlinear system by substitution
2) Solve a nonlinear system by elimination

CAUTION: Non-linear systems cannot be solved using matrices, which are strictly for linear systems.

① $\begin{cases} y=\sqrt{x} & (A) \\ x^2+y^2=6 & (B) \end{cases}$

a) Solve by substitution
b) Solve by elimination
c) Check by graphing

a) Substitution
$y$ is isolated in the first equation (A)
substitute $\sqrt{x}$ for $y$ in the second equation (B)

$x^2+(\sqrt{x})^2=6$ simplify $(\sqrt{x})^2=x$

$x^2+x=6$ recognize quadratic

$x^2+x-6=0$ set $=0$

$(x+3)(x-2)=0$ factor

$x=-3 \quad x=2$ Solve

* GOAL *
Find all ordered pairs where these graphs intersect
• This means for each x, need y-coordinate(s).

$y=\sqrt{-3}$ imaginary is not on graph.
$x=-3$ is an extraneous solution

$y=\sqrt{2}$ is real.

$\boxed{(2,\sqrt{2}) \text{ is the only solution}}$

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① cont.

b) By elimination.

$$\begin{matrix}(A)\\(B)\end{matrix}\quad\left\{\begin{matrix}y=\sqrt{x}\\x^2+y^2=6\end{matrix}\right\}$$

to add these equations and eliminate, we must first square both sides of (A)

(A) $\quad y^2=(\sqrt{x})^2$

$\qquad y^2=x$

Rewrite:

$$\begin{matrix}(A)\\(B)\end{matrix}\quad\left\{\begin{matrix}y^2=x\\x^2+y^2=6\end{matrix}\right.$$

Mult (A) (or B) by $-1$, Add like terms.

$$\begin{array}{rcl}-y^2&=&-x\\x^2+y^2&=&6\\\hline x^2&=&-x+6\end{array}$$

Recognize quadratic

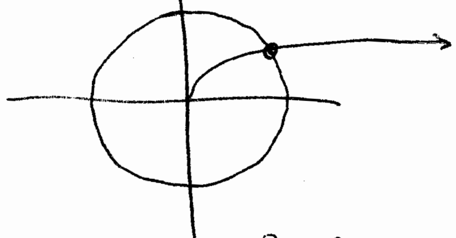
$x^2+x-6=0$

From here, the problem is completed the same as the substitution method.

c) Graphing:

$x^2+y^2=6 \quad\Rightarrow\quad$ circle, center $(0,0)$, radius $\sqrt{6}$

$y=\sqrt{x} \quad\Rightarrow\quad$ half of a right-opening parabola



only one point of intersection

$\sqrt{6}\approx 2.44$

On GC: $\quad x^2+y^2=6 \Rightarrow y^2=6-x^2 \Rightarrow y=\pm\sqrt{6-x^2}$

$y_1=\sqrt{(6-x^2)}$

$y_2=-\sqrt{6-x^2}$

$y_3=\sqrt{x}$

$\boxed{\text{2nd}}\boxed{\text{TRACE}}$ = calc $\quad$ 5. Intersection

Be sure, 1st curve is $y_1$ and 2nd curve is $y_3$!

$\boxed{(2, 1.41)\text{ approx solution}}$

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Alternate Method on GC:

$y_1 = \sqrt{x}$

$y_2 = -x^2+6$ ⇐ hover o =, press enter to turn off

$y_3 = \sqrt{y_2}$ ⇐ $\boxed{\text{VARS}}$ $\boxed{>}$ to Y-VARS $\boxed{\text{ENTER}}$ for Function

$\boxed{\text{ENTER}}$ for Function

$\boxed{\vee}$ to $y_2$

$\boxed{\text{ENTER}}$

$y_4 = -\sqrt{y_2}$

When using 5. Intersect
want $y_1$ to be first curve
and $y_3$ to be 2nd curve

(2) $\begin{cases} x^2 - 3y = 1 & (A) \\ x - y = 1 & (B) \end{cases}$

Substitution: a) Solve (A) for y ⇒ result has fraction $\frac{1}{3}$ ☹
b) Solve (B) for x
c) Solve (B) for y ⇒ must divide/mult by −1.

Option b):

$x - y = 1$

$x = y + 1$

Substitute into (A) ⇒ Use PARENTHESES + FOIL!

$(y+1)^2 - 3y = 1$

$y^2 + 2y + 1 - 3y - 1 = 0$ FOIL

$y^2 - y = 0$ Recognize quadratic, set = 0

$y(y-1) = 0$ Factor + solve.

$y = 0$ or $y = 1$

Substitute back to find x-values: (B) looks easier

$x - 0 = 1$ ⇒ $x = 1$ when $y = 0$ ⇒ $\boxed{(1,0)}$

$x - 1 = 1$ ⇒ $x = 2$ when $y = 1$ ⇒ $\boxed{(2,1)}$

Elimination: cannot eliminate x because x and $x^2$ are not like terms and cannot be combined.

To eliminate y; multiply (B) by −3 to get +3y.

cont →

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(2) con't

$$x^2 - 3y = 1 \quad (A)$$
$$-3x + 3y = -3 \quad (B) \times (-3)$$

$x^2 - 3x = -2$ — write unlike terms side-by-side

$x^2 - 3x + 2 = 0$ — recognize quadratic, set = 0.

$(x-2)(x-1) = 0$ — factor

$x = 2, \quad x = 1$

Substitute back to find y-values: (B) looks easier

$2 - y = 1$
$1 = y$ $\Rightarrow$ $y = 1$ when $x = 2$ $\Rightarrow$ $\boxed{(2,1)}$

$1 - y = 1$
$-y = 0$ $\Rightarrow$ $y = 0$ when $x = 1$ $\Rightarrow$ $\boxed{(1,0)}$
$y = 0$

Graphing

$x^2 - 3y = 1$ — solve for y

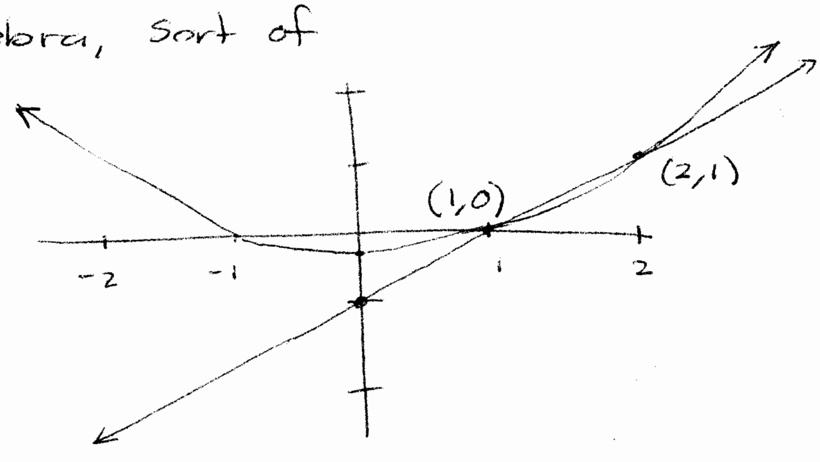
$$-3y = -x^2 + 1$$
$$y = \frac{-x^2}{-3} + \frac{1}{-3}$$
$$Y_1 = \frac{x^2}{3} - \frac{1}{3}$$

$x - y = 1$

$$x - 1 = y$$
$$Y_2 = x - 1$$

Really hard to see! $\boxed{\text{ZOOM}}$ IN (2., then ENTER)

Useful for checking algebra, sort of



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3 $\begin{cases} x^2+y^2=4 & (A) \\ x+y=3 & (B) \end{cases}$

cannot use elimination because $x^2$ and $x$ are not like terms (can't eliminate $x$) and $y^2$ and $y$ are also not like terms (can't eliminate $y$).

Substitution:

solve (B) for either $x$ or $y$. (I choose $y$.)

$$y = 3 - x$$

substitute into (A) — use parentheses and FOIL!!

$$x^2 + (3-x)^2 = 4$$

$$x^2 + 9 - 6x + x^2 = 4$$

$$2x^2 - 6x + 5 = 0$$

recognize quadratic
set = 0

$$(2x \quad 5)(x \quad 1) = 0$$

uh-oh! Doesn't factor!
Quadratic formula or CTS.

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{36-40}}{4}$$

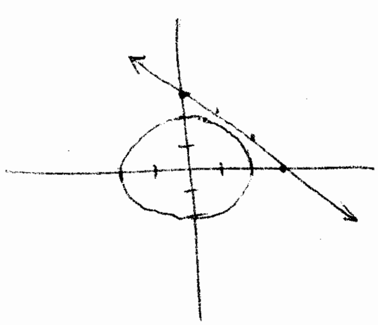
$$x = \frac{6 \pm \sqrt{-4}}{4}$$

$$x = \frac{3}{2} \pm \frac{1}{2} i$$

But wait! These have imaginary parts!
These do not appear on the graph.

$\boxed{\text{NO SOLUTION}}$

Graphing: $x^2+y^2=4$ is circle, center $(0,0)$, radius 2
$x+y=3$ is line $y = -x + 3$



They miss each other just barely —
No intersection,
No Solution.

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(4) $\begin{cases} x^2 + 2y^2 = 10 & \text{(A)} \\ x^2 - y^2 = 1 & \text{(B)} \end{cases}$

Elimination is way easier than solving for x and taking $\sqrt{\ }$...

Multiply (B) by 2 to eliminate y
or (A) or (B) by $-1$ to eliminate x.

I choose (B) by 2:

$$\begin{array}{rcll} x^2 + 2y^2 &=& 10 & \text{(A)} \\ 2x^2 - 2y^2 &=& 2 & \text{(B)} \times 2 \\ \hline 3x^2 &=& 12 & \end{array}$$

combine like terms

$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$x = \pm 2$$

Substitute back $x=2$: into (B)

$$2^2 - y^2 = 1$$
$$4 - y^2 = 1$$
$$4 - 1 = y^2$$
$$3 = y^2$$
$$y = \pm\sqrt{3}$$

$\Rightarrow$ when $x=2$, y is either $\sqrt{3}$ or $-\sqrt{3}$

$\boxed{(2, \sqrt{3}) \quad (2, -\sqrt{3})}$

Substitute back $x = -2$: into (B)

$$(-2)^2 - y^2 = 1$$
$$4 - y^2 = 1$$
$$y = \pm\sqrt{3}$$

$\Rightarrow$ when $x = -2$, y is either $\sqrt{3}$ or $-\sqrt{3}$

$\boxed{(-2, \sqrt{3}) \quad (-2, -\sqrt{3})}$

*CAUTION* Write out all ordered pairs.
Do not use $\pm$ notation, which can be ambiguous. Did you mean 2 pts or 4?

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(4) con't check by graphing

$$x^2 + 2y^2 = 10$$

$$+2y^2 = -x^2 + 10$$

$$y^2 = \frac{-x^2}{+2} + \frac{10}{+2}$$

$$y^2 = \frac{-x^2}{2} + 5$$

$$y = \pm\sqrt{\frac{-x^2}{2} + 5}$$

$$Y_1 = \sqrt{-x^2/2 + 5}$$

$$Y_2 = -\sqrt{-x^2/2 + 5}$$

$$x^2 - y^2 = 1$$

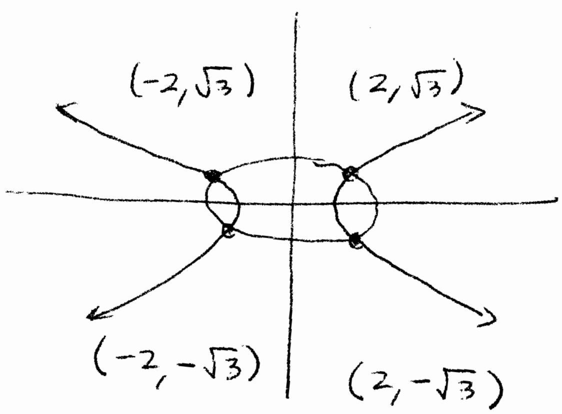
$$-y^2 = -x^2 + 1$$

$$y^2 = x^2 - 1$$

$$y = \pm\sqrt{x^2 - 1}$$

$$Y_3 = \sqrt{x^2 - 1}$$

$$Y_4 = -\sqrt{x^2 - 1}$$



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Extras.

Solve the system.

(6) $\left.\begin{array}{l} x^2+3y^2=6 \\ x^2-3y^2=10 \end{array}\right\}$ $\varnothing$

* (7) $\left.\begin{array}{l} x^2+y^2=25 \\ x=y^2-5 \end{array}\right\}$ $(-5,0)$ $(4,-3)$ $(4,3)$

(8) $\left.\begin{array}{l} 3x^2+y^2=9 \\ 3x^2-y^2=9 \end{array}\right\}$ $(\sqrt{3},0)$ $(-\sqrt{3},0)$

(9) $\left.\begin{array}{l} x=-y^2\cancel{*}-3 \\ x=y^2-5 \end{array}\right\}$ $(-4,-1)$ $(-4,1)$

(10) $\left.\begin{array}{l} x^2+2y^2=4 \\ x^2-y^2=4 \end{array}\right\}$ $(-2,0)$ $(2,0)$.

* (11) $\left.\begin{array}{l} x^2+y^2=1 \\ x^2+(y+3)^2=4 \end{array}\right\}$ $(0,-1)$

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PQ: 10.3

(1) $\left.\begin{array}{l} x^2+y^2=4 \\ x+y=-2 \end{array}\right\}$ $(2,0)$ $(-2,0)$

(2) $\left.\begin{array}{l} x^2+y^2=9 \\ 16x^2-4y^2=64 \end{array}\right\}$ $(-\sqrt{5},-2)$, $(-\sqrt{5},2)$, $(\sqrt{5},-2)$ $(\sqrt{5},2)$

10.3.1 Solve the nonlinear system of equations for real solutions.

$$\begin{cases} x^2 + y^2 = 25 & A \\ 3x + 4y = 0 & B \end{cases}$$

No like terms to eliminate. Use substitution method.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

○ A. ☐
(Simplify your answer. Type an ordered pair. Use a comma to separate answers as needed. Type exact answers for each coordinate, using radicals as needed.)

◉ B. There is no solution.

Solve B for a variable – I choose y.

$$4y = -3x$$

$$y = \frac{-3}{4}x$$

Substitute into A.

$$x^2 + \left(\frac{-3}{4}x\right)^2 = 25$$

Simplify and solve

$$x^2 + \frac{9x^2}{16} = 25$$

$$\frac{16}{16}x^2 + \frac{9}{16}x^2 = 25$$

$$\frac{25}{16}x^2 = 25$$

$$x^2 = \cancel{25} \cdot \frac{16}{\cancel{25}}$$

$$x^2 = 16$$

$$x = \pm 4$$

Substitute x=4:

$$3(4) + 4y = 0$$

$$4y = -12$$

$$y = -3$$

$$\boxed{(4,-3)}$$

Substitute x=-4:

$$3(-4) + 4y = 0$$

$$4y = 12$$

$$y = 3$$

$$\boxed{(-4,3)}$$